# PH210 Logic 2: Metatheory
# Term 1, 2021–2022

### Module leader


- Benedict Eastaugh (Benedict.Eastaugh@warwick.ac.uk)
  Please use your university email and put "PH210" in the subject line.


### Website

https://moodle.warwick.ac.uk/course/view.php?id=47729

Readings, announcements, and problem sets will be posted at this address.

### Module format for 2021–22

1. **Lectures.** Online asynchronous, ± 1.5 hours/week, uploaded on Tuesday.
2. **Problem classes** (*attendance expected*)**.** Monday 12:00–13:00 online synchronous on Teams, starting in week 2.
3. **Drop-in problem sessions** (*attendance optional*)**.**
   - Monday 16:00–17:00 by individual appointment, on Teams or in person (please email for a slot).
   - Monday 17:00–18:00 online synchronous on Teams.

### Description

This module will develop the metatheory of propositional and first-order logic. Our primary goal will be to show that a proof system similar to that presented in Logic 1 is sound (i.e. proves only logically true sentences) and complete (proves all logically true sentences). In order to better understand how we prove things about (as opposed to within) a proof system, we will first study the syntax, semantics, and proof theory of propositional logic. We will then consider Tarski's definitions of satisfaction and truth in a model and proceed to develop the Henkin completeness proof for first-order logic. Other topics covered along the way will include countable versus uncountable sets, the compactness theorem, and the expressive limitations of first-order logic.

PH210 is recommended as a prerequisite for PH340 (Logic 3: Incompleteness and Undecidability), PH341 (Modal Logic), and PH345 (Computability Theory).

### Prerequisites

PH136 (Logic 1) is recommended as a prerequisite. Otherwise, the module is designed to be as self-contained as possible. Some degree of mathematical maturity is helpful, as is a familiarity with elementary set theory and proofs by induction, but neither are strictly required as we will develop the requisite knowledge and proof techniques during the module.

### Reading

The main reading for this module will be a customised version of the Open Logic textbook, which is available on the Moodle page. A selection of background and further reading is available below.

### Problem class

All students are expected to attend and participate in the problem class. This is particularly important for Philosophy students as attendance contributes to the monitoring point system. This is also your opportunity to clarify anything from the lecture or the readings and to get help with exercises.

### Assessment

The module will be assessed (100%) by a two-hour online exam in the summer. You will have 2 hours in which to answer 3 questions (from a choice of 6).

Past exam papers for this module are available, and are a good guide to the type and level of difficulty of the questions in this year's exam. However, please note that the course textbook changed in 2020–2021. Past exam papers prior to that year therefore use slightly different notation and terminology, although the examinable material is similar. Worked solutions to a previous exam paper are available on the Moodle page. There will be a revision session for the module before the start of the exam period.

### How to do well in this module

The lectures will follow our customised version of the Open Logic textbook, as detailed in the schedule below. It will therefore be useful to have read the relevant sections of the textbook before watching the corresponding lectures. Doing the weekly exercises during the term is also essential for building and testing your understanding of the material as we go along. Solutions will be posted on the module website, and discussed in both the problem class and the drop-in problem sessions. Additional support on problem solving techniques is presented in the appendices to Open Logic and in the book *How to Prove It* mentioned above.

### Schedule

The following is an indicative module outline. We may cover a little more or a little less, depending on how things go.

| **Week** | **Topics** | **Textbook** |
|---|---|---|
| 1 | Introduction, inductive definitions, syntax of propositional logic | 1.1–1.2, A, F |
| 2 | Semantics of propositional logic | 1.3–1.6 |
| 3 | Natural deduction for propositional logic | 2, 3 |
| 4 | Completeness for propositional logic | 4 |
| 5 | First-order syntax and semantics | 5 |
| 6 | *Reading week (no lecture or problem class)* | |
| 7 | Examples of theories and models | 6 |
| 8 | Natural deduction for first-order logic | 7 |
| 9 | Completeness for first-order logic | 8 |
| 10 | Applications of completeness, beyond FOL, preview of Logic 3 | 9, 10, 11, 12 |

**Resources**

Here you can find further reading, including other textbooks that might help by giving a slightly different view on the same material. Links to online versions of some texts, as well as library catalogue details, are available on the module reading list.

1. *Supplementary reading*

   (a) *Language, Proof and Logic* (2nd ed.) by Dave Barker-Plummer, Jon Barwise, and John Etchemendy (CSLI Publications, 2011).

   https://webcat.warwick.ac.uk/record=b2533394~S1

   Covers a lot of the basics, in case you want to improve your understanding of those.

   (b) *How to Prove It: A Structured Approach* (2nd ed.) by Daniel J. Velleman (Cambridge University Press, 2006).

   http://webcat.warwick.ac.uk/record=b2484668~S1 (ebook)

   Covers basic proof techniques, especially recommended for those with a strong mathematical background.

   (c) *The Mathematics of Logic: A Guide to Completeness Theorems and Their Applications* by Richard Kaye (Cambridge University Press, 2007).

   https://webcat.warwick.ac.uk/record=b2521388~S1 (ebook)

   Mathematical approach to propositional and first-order logic, focusing on completeness theorems and their combinatorial and topological properties.

2. *Other textbooks*

   (a) *Logic and Structure* (5th ed.) by Dirk van Dalen (Springer, 2013).

   https://webcat.warwick.ac.uk/record=b2773545~S1 (ebook)

   A mature and polished textbook on the metatheory of propositional and first-order logic. Contains additional material on second-order and intuitionistic logic if you would like to explore further topics.

   (b) *A Mathematical Introduction to Logic* (2nd ed.) by Herbert B. Enderton (Harcourt/Academic Press, 2001).

   https://webcat.warwick.ac.uk/record=b3598185~S1 (ebook)

   Classic textbook by a master of the genre. Aimed more at mathematicians, but accessible to philosophers.

   (c) *Mathematical Logic* (3rd ed.) by Hans-Dieter Ebbinghaus, Jörg Flum, and Wolfgang Thomas (Springer, 2021).

   https://webcat.warwick.ac.uk/record=b3517402~S1

   Updated version of a classic. An advanced textbook which covers a number of topics beyond the scope of this course, including infinitary and second-order logic, decidability and undecidability, logic programming, and Lindström's theorem.