PH342 Philosophy of Mathematics
Term 2, 2020–2021

Module leader

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  Please put "PH342" in the subject line of your email and sign your full name.

  Advice and feedback hours take place on Wednesdays, 10:00–12:00. Book a slot here, or email me for an appointment.

Website

https://moodle.warwick.ac.uk/course/view.php?id=41252

Announcements and other materials will be posted at this address.

Description

This course is a first introduction to philosophy of mathematics, via one of our most fascinating and perplexing concepts: *the infinite*. We encounter the concept of infinity in myriad ways. In Zeno's paradoxes of time, space, and motion, the idea of infinite division is used to argue in favour of a radical monism. The ancient atomists Leucippus and Democritus claimed that the universe consisted of an infinity of atoms moving in an infinite void, and contemporary cosmology still considers the issue of whether the universe is infinite to be an open question.

But what does it mean for something to be infinite? It is mathematics that offers us the precise definitions that let us begin to answer this question, and thus in mathematics that many of the most important questions concerning the infinite arise. Do the infinite structures that we talk about in mathematics really exist? If so, how can we have knowledge of them? Is it even coherent to talk about the truly infinite, or does it fall victim to paradox? This course will investigate these and other questions by engaging with the ideas of philosophers and mathematicians from across history, with a focus on the reception of Georg Cantor's theory of sets, and the crisis in the foundations of mathematics that it precipitated.

Prerequisites

PH136 (Logic 1) is recommended as a prerequisite. Otherwise, the module is designed to be as self-contained as possible. But you should be aware that several of the topics we will discuss are related to developments in mathematical logic (as treated in modules like Logic 2/3 and Set Theory) and also build on philosophical themes from metaphysics, epistemology, and the philosophy of language. A background in these subjects will therefore be helpful in fully engaging with the module content.

How the module works

There are two types of synchronous (live) events that you are expected to attend:

- *Online synchronous lectures*: Mondays 12:00, weeks 2–5 and 7–10, on Microsoft Teams.
- *Seminars*:
  - Mondays 17:00–18:00.
  - Mondays 18:00–19:00.
  - Mondays 19:00–20:00.
  - Tuesdays 19:00–20:00.

  Seminars start in week 2, and will take place online on Microsoft Teams.

In addition to these live events, I will be releasing **pre-recorded lectures** every Thursday. An announcement will be sent once the videos are uploaded, so you don't need to keep checking the website for updates.

The lectures are intended to give you a broad introduction to the topics we are covering. In the seminars we will have more focused discussions of those topics, making reference to the core readings for that week. Seminars will be devoted to the topics presented in the pre-recorded lectures released the previous Thursday, so before your seminar you are expected to have done the following:

1. Watched the pre-recorded lectures from the previous Thursday.
2. Attended the live lecture.
3. Read the core assigned texts.

Core texts

Much of the background and historical reading will be drawn from two books:

- *The Infinite* (2nd ed.) by A. W. Moore (Routledge, 2001).
  https://pugwash.lib.warwick.ac.uk/record=b2893101~S1
- *The Search for Certainty* by M. Giaquinto (Oxford University Press, 2002).
  https://pugwash.lib.warwick.ac.uk/record=b2324098~S1

There will also be a substantial use of original sources and recent scholarship. Many important papers can be found in the following collections.

- *Philosophy of Mathematics: Selected Readings* (2nd ed.), edited by P. Benacerraf and H. Putnam (Cambridge University Press, 1983).
  https://pugwash.lib.warwick.ac.uk/record=b2797754~S1
- *From Frege to Gödel: A Source Book in Mathematical Logic, 1879–1931*, edited by J. van Heijenoort. https://webcat.warwick.ac.uk/record=b1350155~S1

An excellent handbook in philosophy of mathematics is:

- *The Oxford Handbook of Philosophy of Mathematics and Logic*, edited by S. Shapiro (Oxford University Press, 2005). https://webcat.warwick.ac.uk/record=b2663883~S1

A survey of topics in mathematical logic that are relevant for this course is:

- *Mathematical Logic* by J. R. Shoenfield (Addison-Wesley, 1967).
  http://webcat.warwick.ac.uk/record=b3517405~S1

Assessment

Assessment for this module is based on a two-hour exam (80%) as well as a 1000-word essay (20%). The deadline for the essay is Tuesday of Week 7 (23 February 2021). Due to the current situation, you can self-certify for a one-week extension to this deadline without providing a justification, provided that you self-certify *prior to the deadline*.

More details and guidance on the assessed essay and the exam appear below.

Schedule and list of readings

- **Week 1**: Introduction to the module. The infinite in ancient Greek thought.
  - *Core reading*: introduction and chapters 1 and 2 of Moore [2001].
  - *Supplementary reading*:
    - Early Greek thought: Curd [2020], Huffman [2018, 2019].
    - Zeno's paradoxes: Huggett [2019], Palmer [2021, 2020].
    - Ancient Greek atomism: Berryman [2016a,b,c].
    - Aristotle on infinity: Mendell [2019], sections 1 and 2 of Linnebo and Shapiro [2019], Hintikka [1966], Lear [1980].
- **Week 2**: Infinity in mathematics. Cantor's theory of sets.
  - *Core reading*: part I (pages 1–34) of Giaquinto [2002].
  - *Supplementary reading*:
    - Bolzano/Dedekind and visual proofs in analysis: chapters 4 and 8 of Moore [2001], Russ [2004], preface to 'Continuity and Irrational Numbers' in Dedekind [1963], Reck [2012], Giaquinto [2020].
    - Early development of set theory: chapter 8 of Moore [2001], chapters 1–5 of Dauben [1979], introduction and chapter 1 of Hallett [1984], chapters I–VI of Ferreirós [2007], Ferreirós [2020].
- **Week 3**: The class-theoretic paradoxes. Type theory and limitation of size.
  - *Core reading*: chapter 10 of Moore [2001], part II (pages 35–65) of Giaquinto [2002].
  - *Supplementary reading*:
    - Frege and logicism: Demopoulos and Clark [2005], Tennant [2017].
    - Class-theoretic paradoxes: Irvine and Deutsch [2020], Mortensen [2017].
    - Limitation of size: chapters 4–5 of Hallett [1984], chapters 6–11 of Dauben [1979].
    - Russell and type theory: Linsky and Irvine [2020], Coquand [2018], Gödel [1944].
- **Week 4**: The axiomatic method. Hilbertian finitism.
  - *Core reading*: Chapters IV.3–4 of Giaquinto [2002], Hilbert [1926] (up to page 384 in van Heijenoort [1967]).
  - *Supplementary reading*:
    - The axiomatic method: Torretti [2019].
    - The Frege–Hilbert controversy: Blanchette [2018].
    - Hilbert's finitism: Detlefsen [2005], Bernays [1930], Zach [2009].
    - Hilbert's program: Franks [2017], Zach [2009, 2006].
    - Incompleteness: Franzén [2005], Smoryński [1977].

- **Week 5**: Constructivism in Brouwer and Heyting.
  - *Core reading*: chapter 7 of Shapiro [2000], chapter 1 ('Disputation') of Heyting [1971].
  - *Supplementary reading*: Brouwer [1912, 1949], van Atten [2017], Bridges and Palmgren [2018], Dummett [1973].
- **Week 6**: **Reading week** (no lectures or seminars)
- **Week 7**: **1000 word essay due** (Tuesday 23 February)

  The Löwenheim–Skolem theorem. Skolem's paradox.
  - *Core reading*: Chapters IV.1 and IV.2 of Giaquinto [2002], and Benacerraf and Wright [1985].
  - *Supplementary reading*: Chapter 11 of Moore [2001], Skolem [1922], Putnam [1980], Hallett [2011], Bays [2014].
- **Week 8**: The continuum problem. Realism and indeterminacy in set theory.
  - *Core reading*: Chapter VI.1 of Giaquinto [2002], and Gödel [1947].
  - *Supplementary readings*: Maddy [1988], Koellner [2009], Kanamori [1996], Feferman et al. [2000], Hamkins [2012], Steel [2014], Martin [2001].
- **Week 9**: Categoricity and determinacy. Structuralism.
  - *Core reading*: Chapter 10 of Shapiro [2000], Benacerraf [1965].
  - *Supplementary reading*: Hellman [2005], Parsons [1990], Benacerraf [1996], Shapiro [1997], Reck and Price [2000], Reck [2003], Chihara [2004], Resnik [2019], Reck and Schiemer [2020].
- **Week 10**: Potential infinity revisited. Modality and potentiality.
  - *Core reading*: Linnebo and Shapiro [2019].
  - *Supplementary reading*: Linnebo [2013], Linnebo, Shapiro, and Hellman [2016], Hamkins and Linnebo [2019].

Revision sessions

There will be two revision sessions for the module before the start of the exam period.

- **Revision session 1**: Thursday 29 April 2021.

  This session will explain the details of the exam format. We will then go over the main topics of the module, with an emphasis on what you will need to know for the exam. We will then have time for questions.
- **Revision session 1**: Thursday 13 May 2021.

  This session will focus on good exam technique, including preparing for the exam, and what constitutes a good answer. We will then have time for questions.

Assessed essays

The general guidelines for writing your assessed essay are as follows.

- Your essay should be 1000 words (± 10%).
- The submission deadline is 15:00 GMT on Tuesday 23 February 2021.
- Essays are to be submitted via Tabula.
- Submission instructions can be found here.
- Marking criteria can be found here.

You can find some help in writing a good philosophy essay in the following guides.

- Writing philosophy essays, by Neil Dewar (but ignore the marking guidelines!).
- How to write a philosophy paper, from the Pink Guide to Philosophy.
- Guidelines on writing a philosophy paper, by Jim Pryor.

The Philosophy department also has writing support available.

- Book a writing support session.

The list of pre-approved essay titles is as follows. If you would like to write on a different topic, please email me with your proposal and we can discuss it.

Pre-approved essay titles

1. Is an Aristotelian denial of actual infinity tenable in the light of our use of infinitary notions such as limits in mathematical analysis?
2. Were 19th century analysts like Bolzano and Dedekind correct to reject the use of spatial intuition or geometrical reasoning in analytical proofs?
3. How thoroughly do the class paradoxes, such as Russell's paradox, undermine the possibility of a logicist foundation for mathematics?
4. In what sense is Russellian type theory a logicist approach to the foundation of mathematics?
5. To what extent are formalist approaches to mathematics free of the use of intuition? Assess with reference to Hilbert and Brouwer's use of Kantian ideas.
6. Explain the aims of Hilbert's program. Was it defeated by Gödel's incompleteness theorems?
7. Are non-constructive principles such as the Law of Excluded Middle valid principles of mathematical reasoning? Explain your answer with reference to Brouwer's objections to classical logic.
8. Explain and assess Dummett's claim that an intuitionistic interpretation of mathematical statements can only be defended on the basis of semantic considerations.

Exam

The module will be assessed (80%) via a two-hour online exam in the summer, via the Alternative Exams Portal. The exam format will be substantially the same as in previous years: you will need to answer 2 essay questions (out of 6). There is a 2500-word overall word limit. For more details please see the link below.

- Philosophy exam guidelines: essay-based exams in 2020–2021.

Past exam papers for this module are available, and are a good guide to the type and level of difficulty of the questions in this year's exam. However, since the module content has changed somewhat this year, please be aware that past exam papers do not offer a good guide to the content of this year's exam. Examinable topics are restricted to those which we have studied in the module.

- Past exam papers for PH342.

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